Sémantique des jeux

du λ-calcul à la logique classique du 1^{er} ordre

LAC – Chambéry

Olivier LAURENT

Olivier.Laurent@pps.jussieu.fr

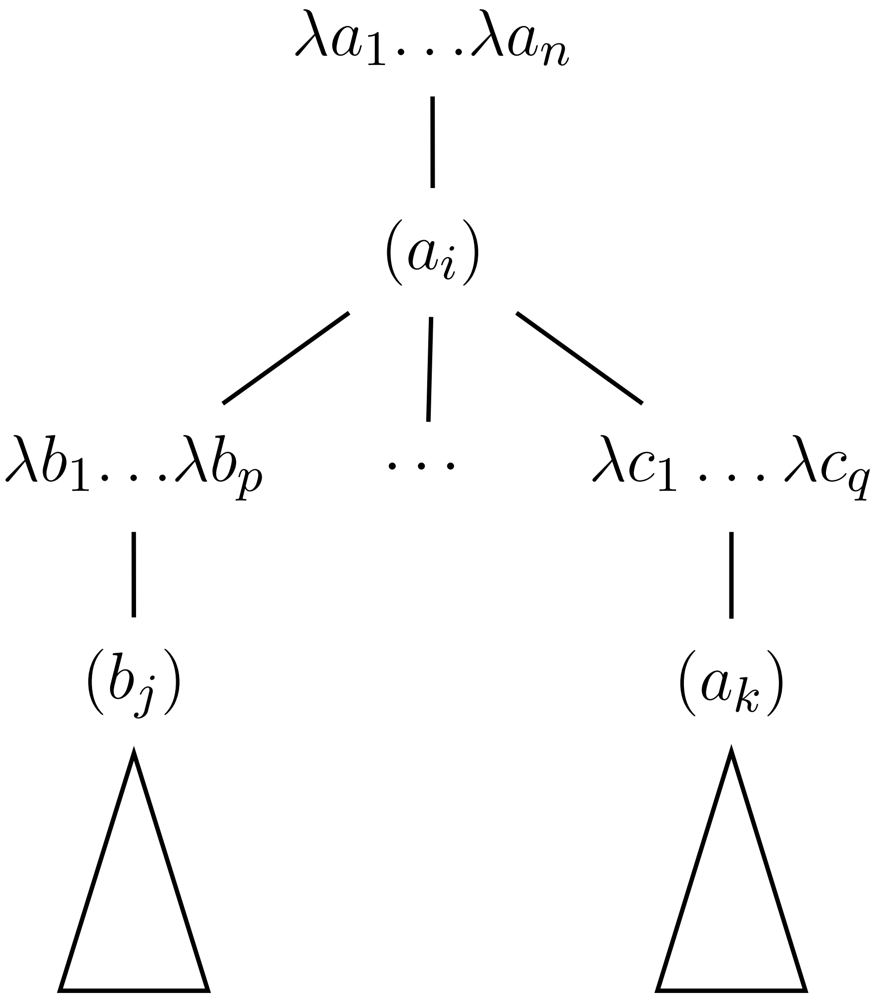
Preuves Programmes Systèmes

CNRS – Université Paris VII

De la syntaxe à la sémantique

Calcul des séquents	Déduction naturelle	λ-calculs	Jeux HO/N
LJ	NJ	λ	innocence parenthésage
LK	NK	$\lambda\mu$	innocence
élimination des coupures	normalisation	β-réduction	composition

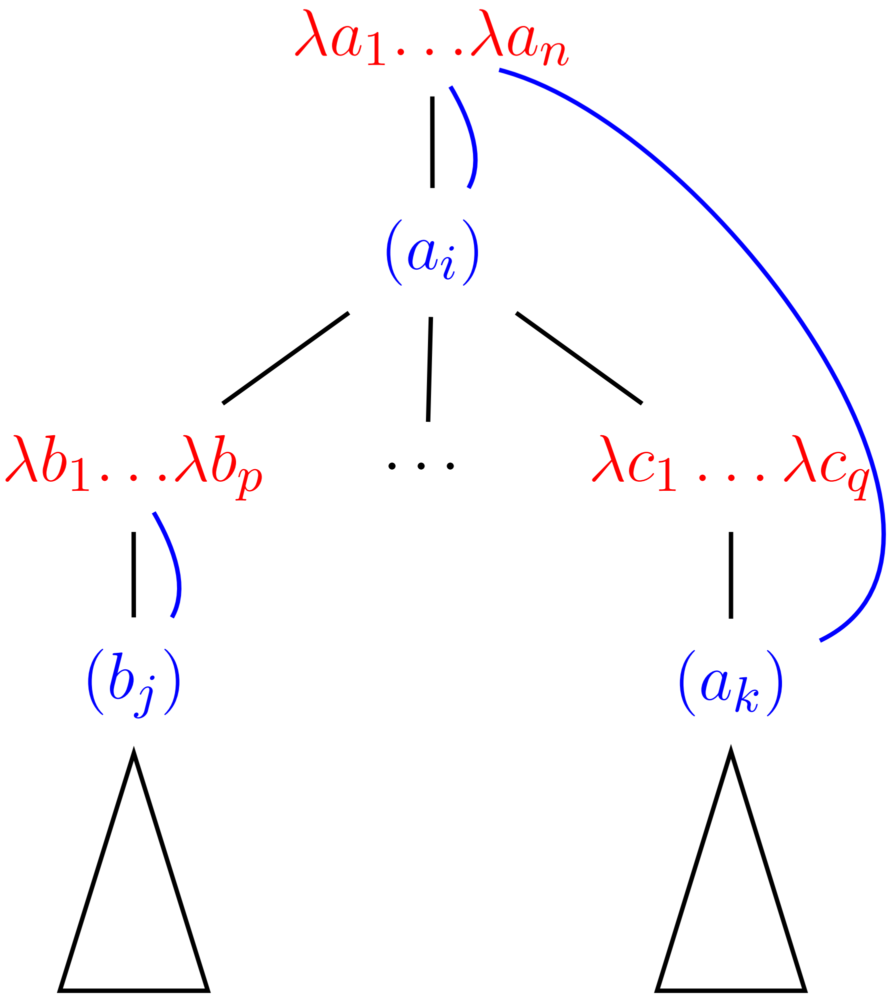
Arbre de Böhm



$$A_1 \to \cdots \to A_n \to o$$

$$B_1 \to \cdots \to B_m \to o \quad (= A_i)$$

Arbre de Böhm



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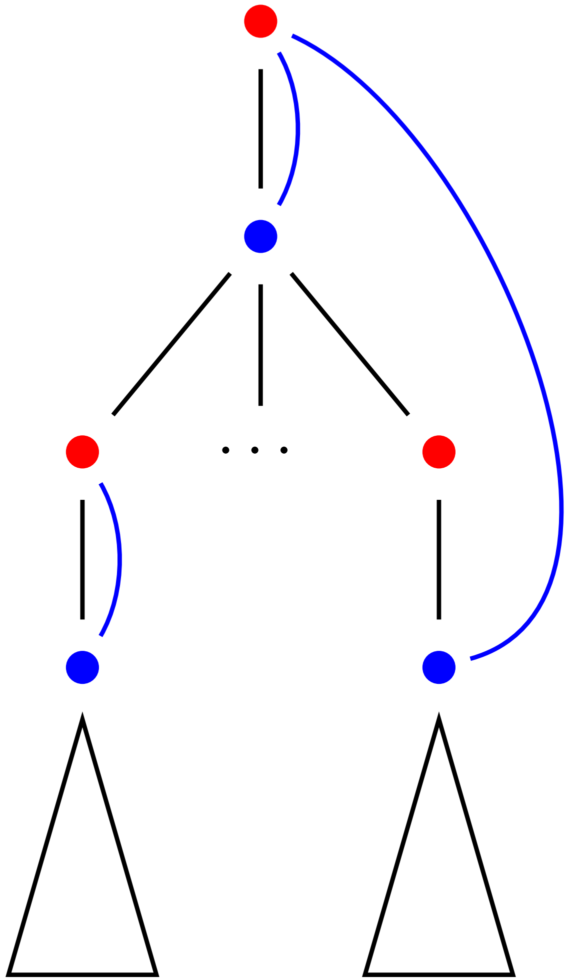
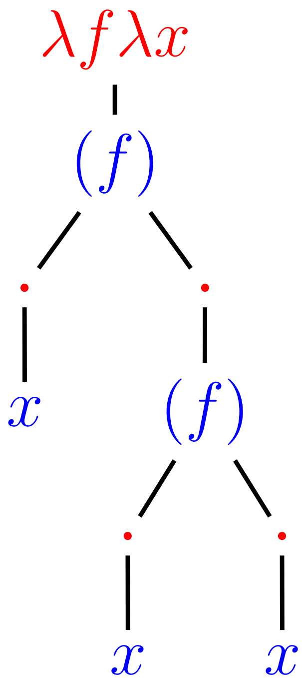
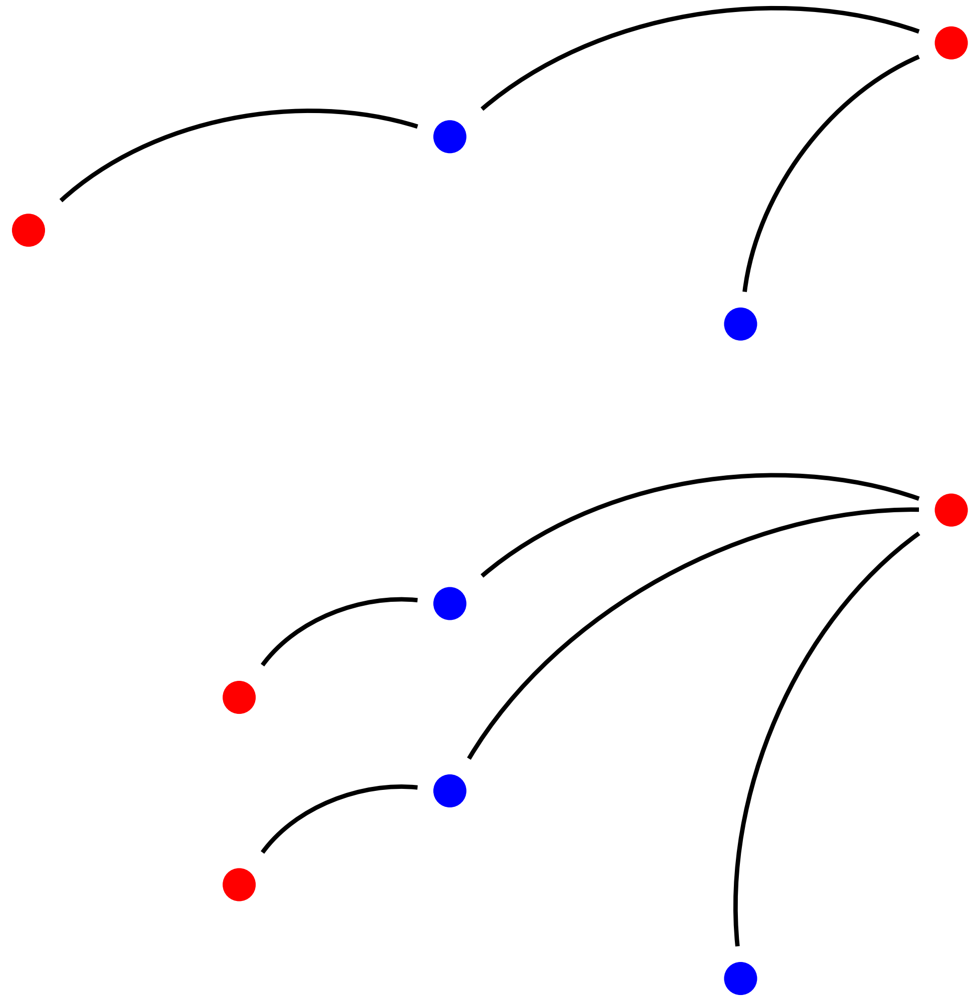


Stratégies

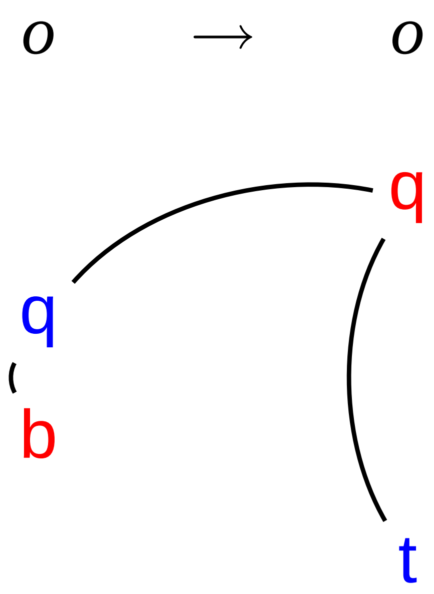
$\lambda f.\lambda x.(f)x(f)xx$

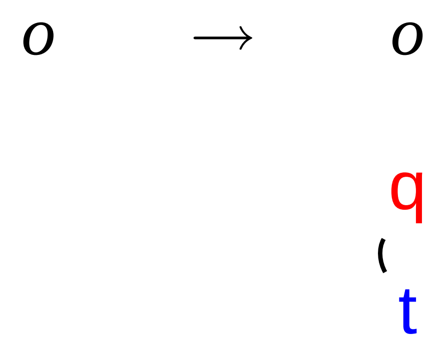
$(\ o \rightarrow o \rightarrow o\ ) \rightarrow o \rightarrow o$





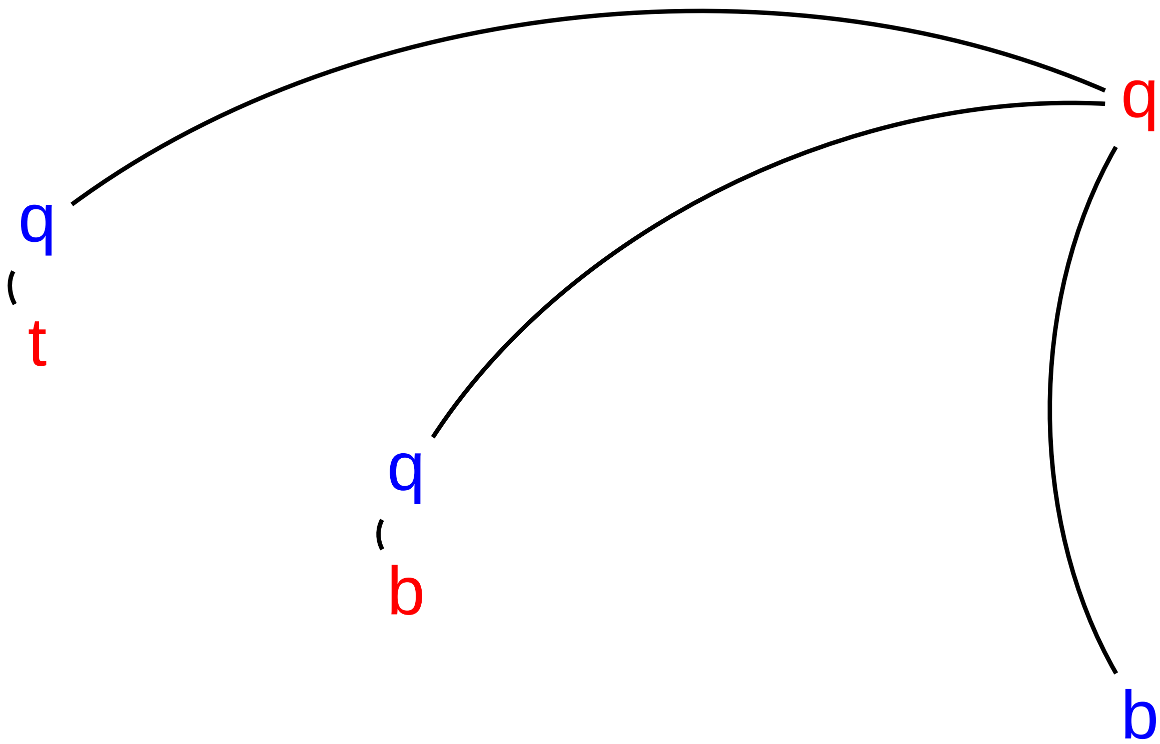
Stratégies avec booléens (constantes)





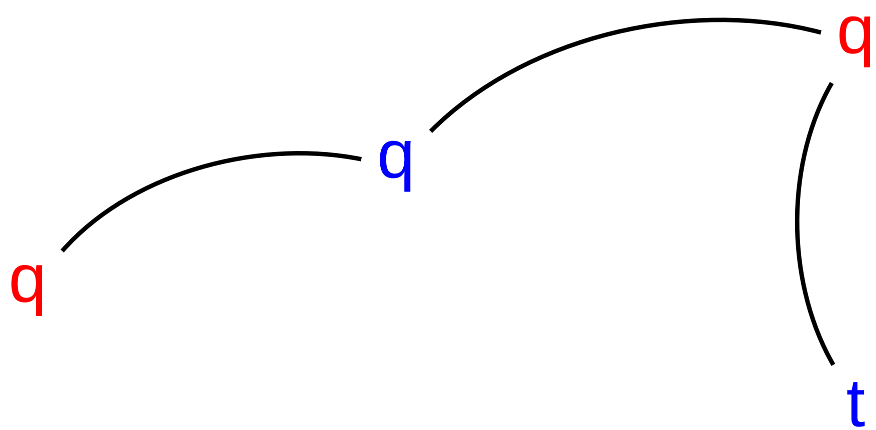
Stratégies avec booléens (case)

$$o \quad \rightarrow \quad o \quad \rightarrow \quad o \quad \rightarrow \quad o$$



Stratégies avec booléens (`catch`)

$$(\; o \; \longrightarrow \; o \;) \; \longrightarrow \; o$$



λ-calcul avec constantes

Types et termes

$$A ::= A \to A \mid o$$

$$t ::= a \mid \lambda a.t \mid (t)t \mid \mathtt{c}_i \mid \mathtt{case}\ t\ \mathtt{of}\ \overrightarrow{\mathtt{c}_i \mapsto t}$$

Typage

$$\frac{}{\Gamma \vdash \mathtt{c}_i : o}\ \mathtt{c}_i \qquad \frac{\Gamma \vdash t : o \qquad \cdots\ \Gamma \vdash t_i : A\ \cdots}{\Gamma \vdash \mathtt{case}\ t\ \mathtt{of}\ \overrightarrow{\mathtt{c}_i \mapsto t_i} : A}\ \mathtt{case}$$

Simplification des réponses

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-

$\vdots$

-
- $\quad : o$

Simplification des réponses

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$: o$

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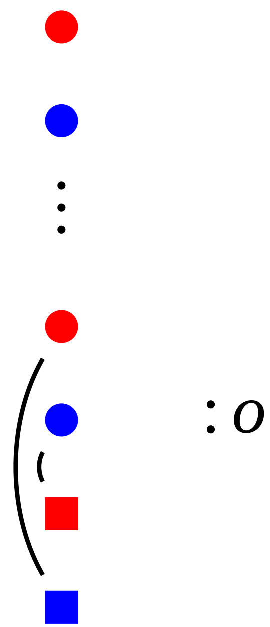
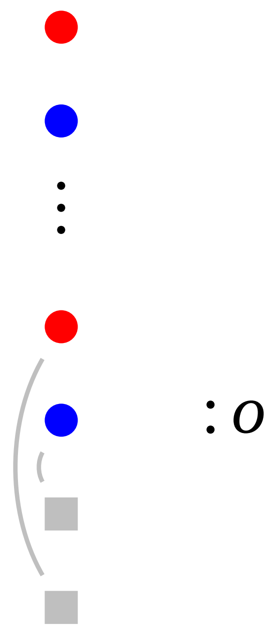
Simplification des réponses

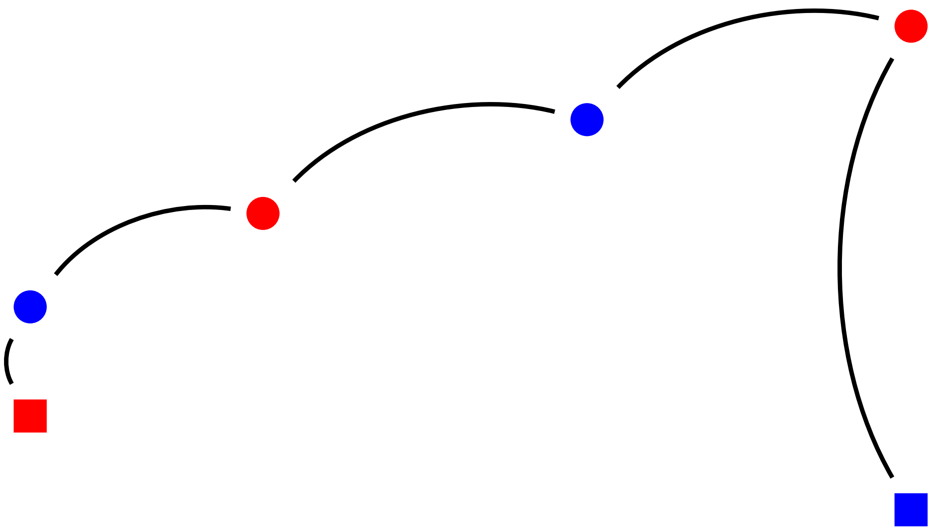


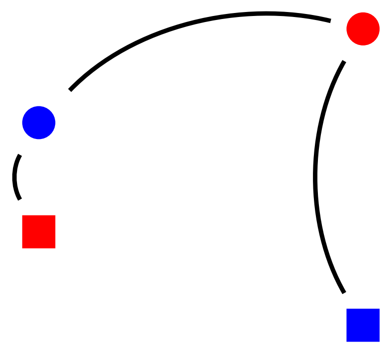
Simplification des réponses



`call/cc` et loi de Peirce

$$((o \to o) \to o) \to o$$





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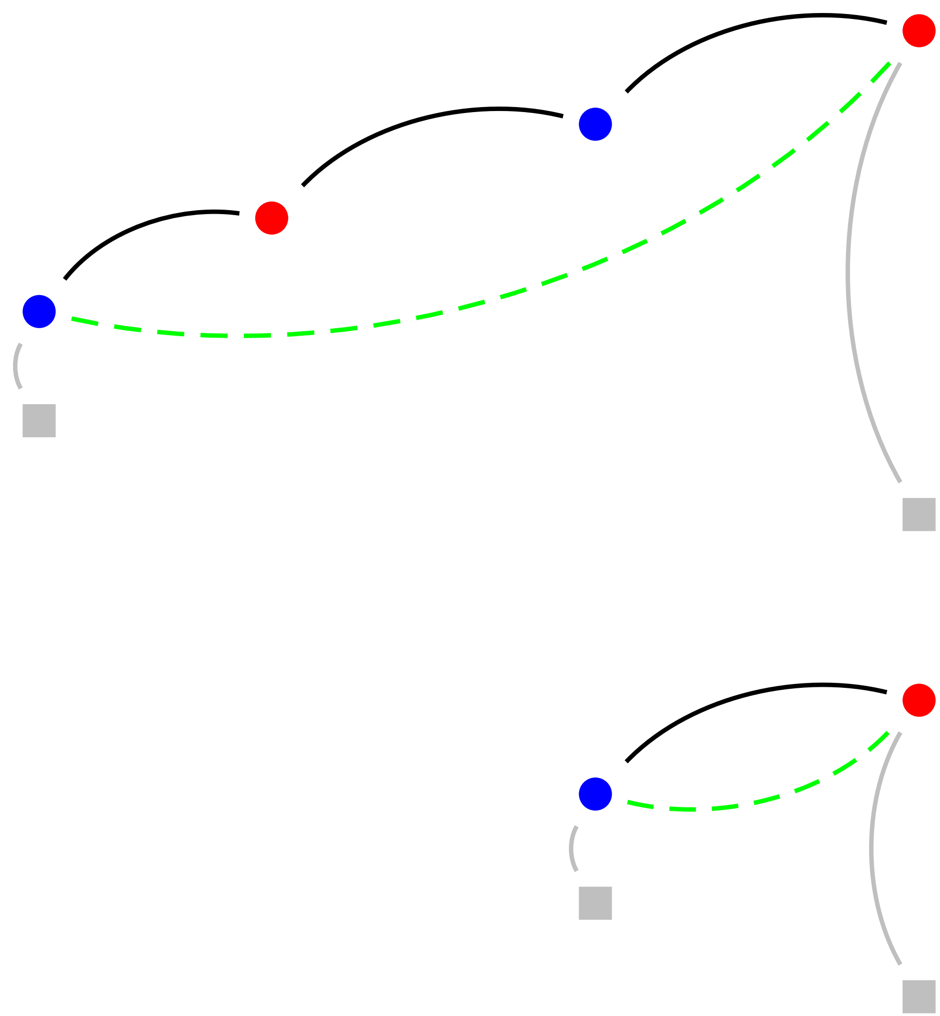

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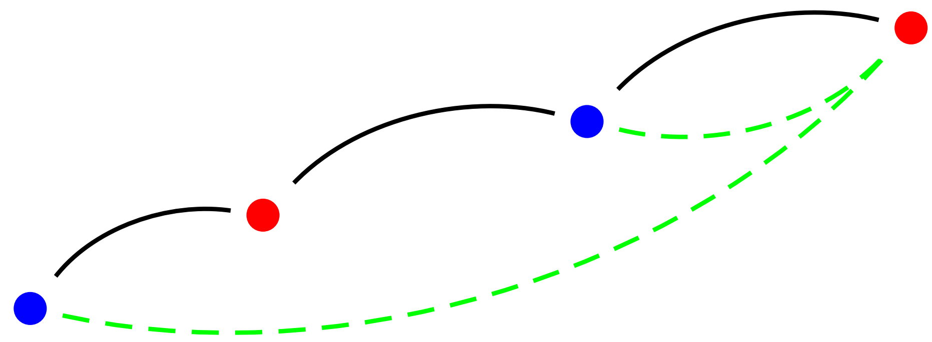
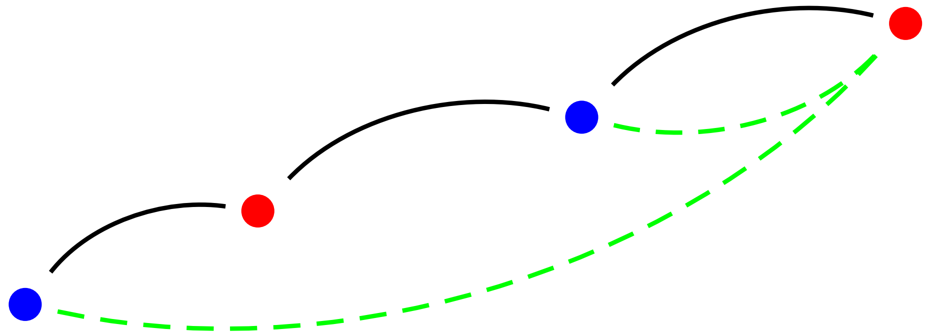
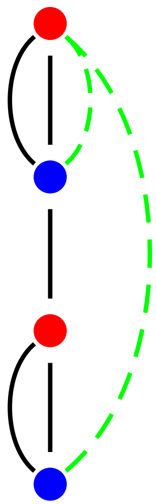


`call/cc` et loi de Peirce

$$((o \rightarrow o) \rightarrow o) \rightarrow o$$





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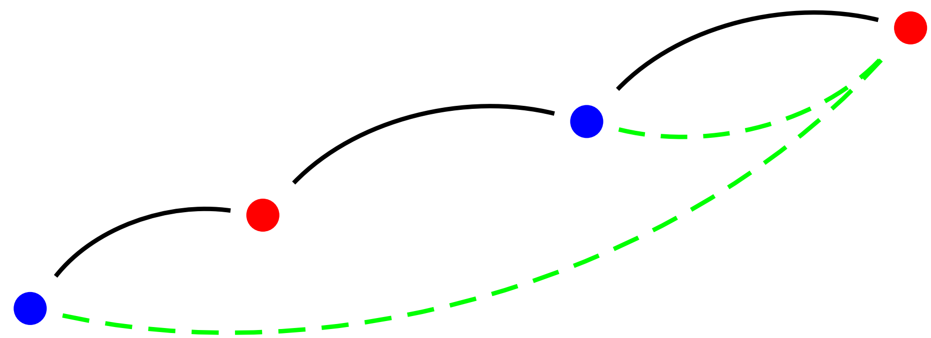

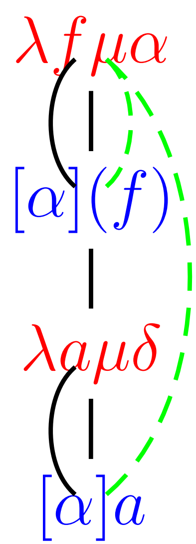

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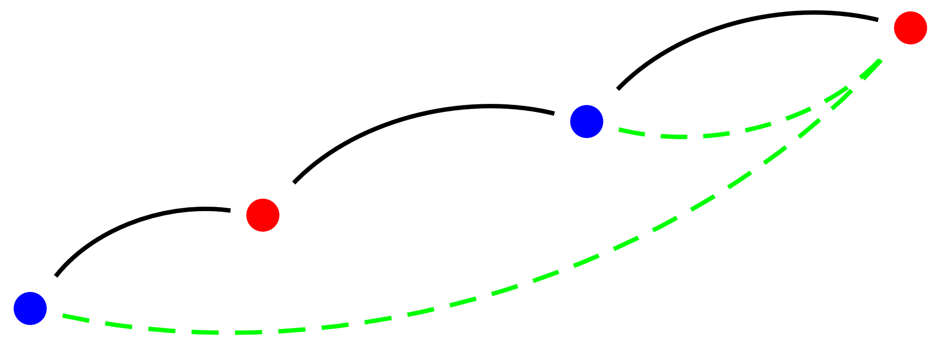
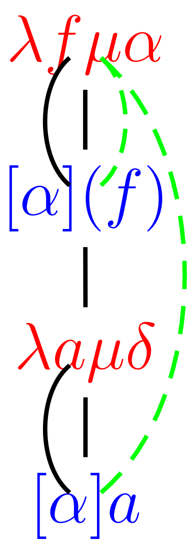
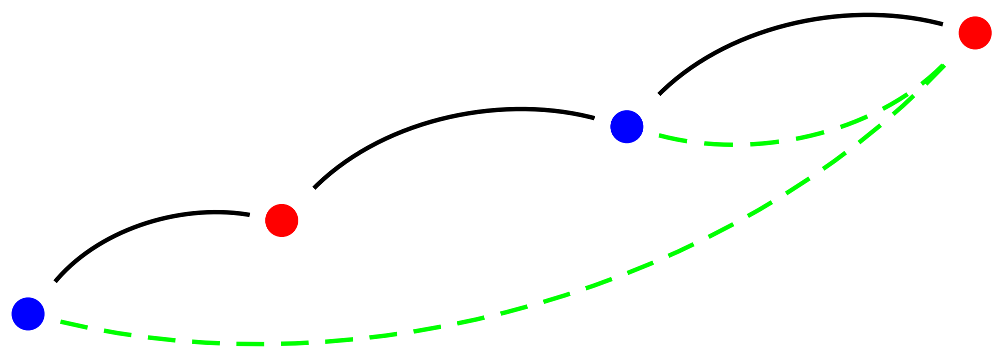


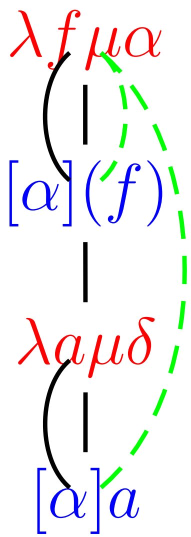


$$\lambda f.\mu\alpha[\alpha](f)\lambda a.\mu\delta[\alpha]a$$

call/cc et loi de Peirce

$$((X \rightarrow Y) \rightarrow X) \rightarrow X$$





$$\lambda f.\mu\alpha[\alpha](f)\lambda a.\mu\delta[\alpha]a$$

Logique du 1er ordre : le buveur

$$\exists x \forall y (Xx \to Xy)$$

$$|||$$

$$\forall x \quad (\quad \forall y \quad (\quad Xx \quad \to \quad Xy \quad) \quad \to \quad \bot \quad) \quad \to \quad \bot$$

Logique du 1$^{\text{er}}$ ordre : le buveur

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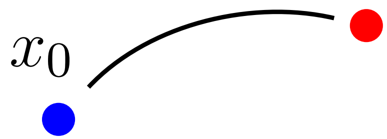

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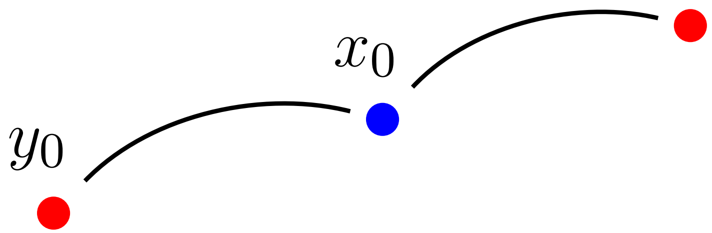
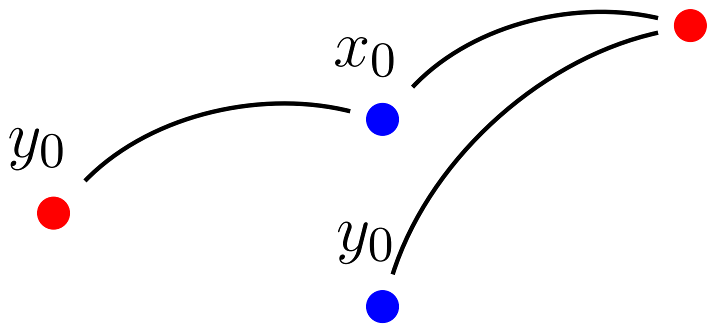


Logique du 1^er ordre : le buveur

$$\exists x \forall y (Xx \rightarrow Xy)$$

$$|||$$

$$\forall x \quad (\quad \forall y \quad (\quad Xx \quad \rightarrow \quad Xy \quad) \quad \rightarrow \quad \bot \quad) \quad \rightarrow \quad \bot$$



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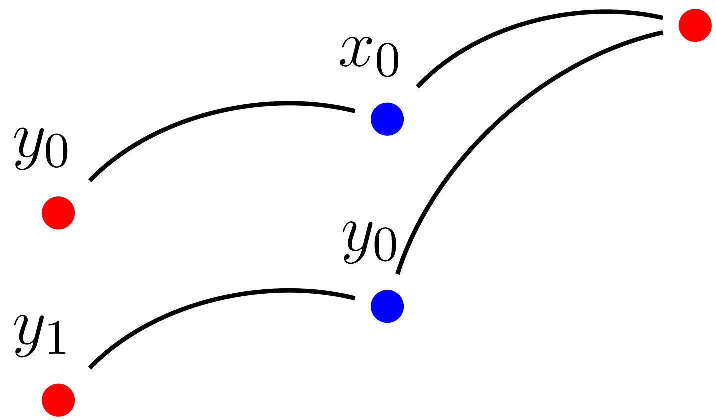

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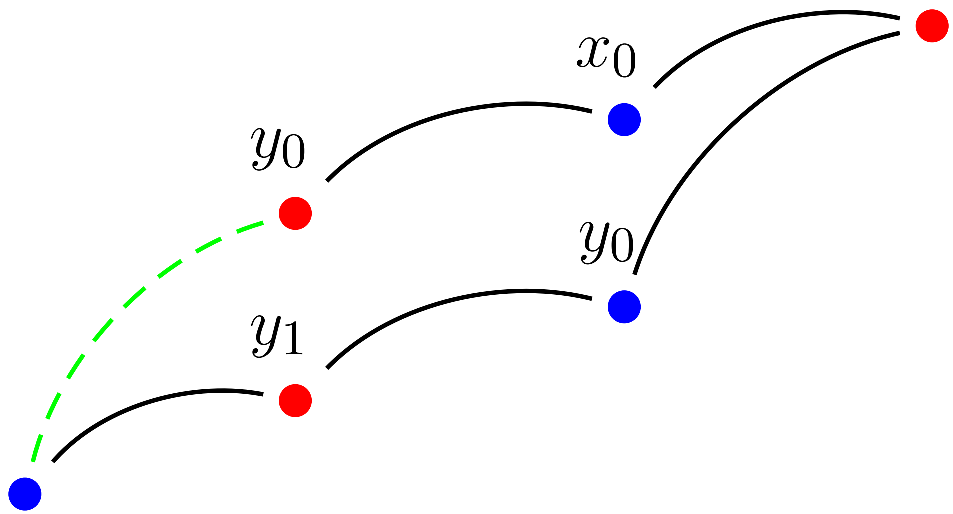

Logique du 1^er^ ordre : le buveur

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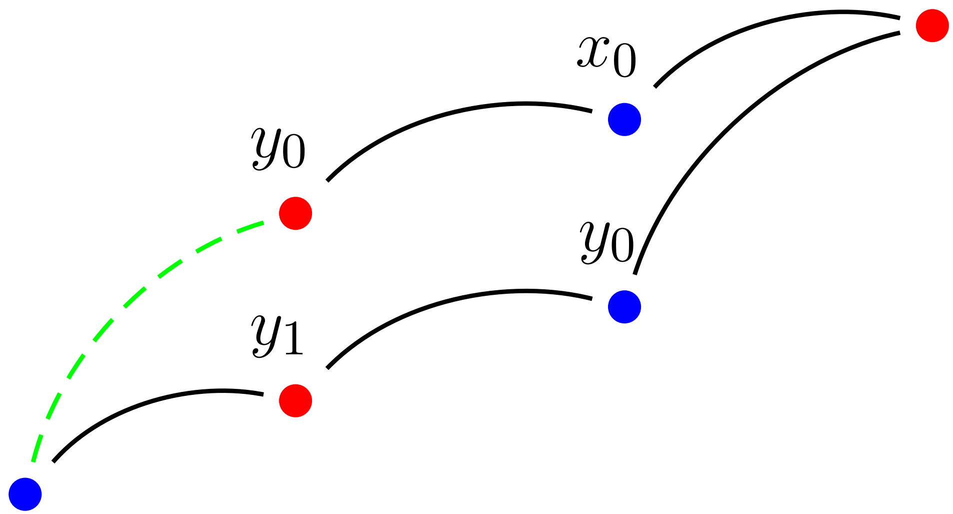

$$\lambda f. \, (f\{x_0\}) \, \Lambda y_0.\lambda d.\mu\alpha. \, (f\{y_0\}) \, \Lambda y_1.\lambda a.\mu\delta \, [\alpha]a$$